Metric Spaces

Decide whether or not each of the following pairs (X, d) are metric spaces. Simply write "Yes" or "No." You do not need to give any proofs or make any arguments at this time.

In 1-5, $a, b \in \mathbb{R}$ are real numbers.

1. (X, d) where $X = \mathbb{R}$ and $d(a, b) = 1$ if $a \neq b$ and $d(a, b) = 0$ if $a = b$

yes

2. (X, d) where $X = \mathbb{R}$ and $d(a, b) = |a^2 - b^2|$

no

3. (X, d) where $X = \mathbb{R}$ and $d(a, b) = |a^3 - b^3|$

yes

4. (X, d) where $X = \mathbb{R}$ and $d(a, b) = \max(a, b)$

no

5. (X, d) where $X = \mathbb{R}$ and $d(a, b) = \max(|a|, |b|)$

no

In 6-10, $a, b \in \mathbb{R}^2$ have the form $a = (x_1, y_1)$ and $b = (x_2, y_2)$.

6. (X, d) where $X = \mathbb{R}^2$ and $d(a, b) = \max(|x_1 - x_2|, |y_1 - y_2|)$

yes

7. (X, d) where $X = \mathbb{R}^2$ and $d(a, b) = (x_1 - x_2)^2 + (y_1 - y_2)^2$

no

8. (X, d) where $X = \mathbb{R}^2$ and $d(a, b) = \sqrt[3]{(x_1 - x_2)^3 + (y_1 - y_2)^3}$

no

9. (X, d) where $X = \mathbb{R}^2$ and $d(a, b) = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ if a and b lie on the same line through the origin and $d(a, b) = \sqrt{(x_1)^2 + (y_1)^2} + \sqrt{(x_2)^2 + (y_2)^2}$ otherwise

yes

10. (X, d) where $X = \mathbb{R}^2$ and $d(a, b) = 1$

no