

Continuity

1. Prove $\lim_{x \rightarrow 3} 2x - 1 = 5$. Is $2x - 1$ continuous at $x = 3$?

2. Prove $\lim_{x \rightarrow 1} f(x) = -1$ where $f(x) = \begin{cases} x - 2 & \text{if } x \geq 1 \\ x^2 - x - 1 & \text{if } x < 1 \end{cases}$

Is f continuous at $x = 1$?

3. Prove $\lim_{x \rightarrow -1} f(x) = 1$ where $f: \mathbb{R} \setminus \{-1\} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x^2 + 3x + 2}{x + 1}$.

How could you extend f to a function on \mathbb{R} that is continuous at $x = -1$?

4. Prove that $\sin x$ is continuous on \mathbb{R} .

Hint: You will need the trig identity $\sin(\alpha + \beta) - \sin(\alpha - \beta) = 2 \cos \alpha \sin \beta$, as well as the facts that $|\cos \alpha| \leq 1$ and $|\sin \alpha| \leq |\alpha|$.