Sequences and Subsequences 1

Topic 1: Convergence

1. Does $a_n = \frac{16}{n^3}$ converge in $\mathbb{R}$? If yes, find what it converges to.

2. Does $a_n = \frac{80}{\sqrt{5n}}$ converge in $\mathbb{R}$? If yes, find what it converges to.

3. Does $a_n = \cos(n)$ converge in $\mathbb{R}$? If yes, find what it converges to.

4. Does $a_n = 1 - \frac{(-1)^n}{n}$ converge in $\mathbb{R}$? If yes, find what it converges to.

5. Does $a_n = \frac{9-7n}{8+13n}$ converge in $\mathbb{R}$? If yes, find what it converges to.

6. Does $a_n = -n$ converge in $\mathbb{R}$? If yes, find what it converges to.

7. Does $a_n = \frac{(1+2n)^2}{5+3n+3n^2}$ converge in $\mathbb{R}$? If yes, find what it converges to.

Topic 2: Boundedness

8. Is $a_n = \frac{16}{n^3}$ bounded? If yes, find a ball (interval) that bounds it.

9. Is $a_n = \frac{80}{\sqrt{5n}}$ bounded? If yes, find a ball (interval) that bounds it.

10. Is $a_n = \cos(n)$ bounded? If yes, find a ball (interval) that bounds it.

11. Is $a_n = 1 - \frac{(-1)^n}{n}$ bounded? If yes, find a ball (interval) that bounds it.

12. Is $a_n = \frac{9-7n}{8+13n}$ bounded? If yes, find a ball (interval) that bounds it.

13. Is $a_n = -n$ bounded? If yes, find a ball (interval) that bounds it.

14. Is $a_n = \frac{(1+2n)^2}{5+3n+3n^2}$ bounded? If yes, find a ball (interval) that bounds it.

Sequences and Subsequences 2

Prove that the following sequences in $\mathbb{R}$ converge.

1. $a_n = \frac{16}{n^3} \to 0$

2. $a_n = \frac{80}{\sqrt{5n}} \to 0$

3. $a_n = 1 - \frac{(-1)^n}{n} \to 1$

4. $a_n = \frac{9-7n}{8+13n} \to \frac{-7}{13}$

5. $a_n = \frac{(1+2n)^2}{5+3n+3n^2} \to \frac{4}{3}$

6. Theorem: If a sequence $\{a_n\}$ converges, then $\{a_n\}$ is bounded.

Converse: If a sequence $\{a_n\}$ is bounded, then $\{a_n\}$ converges.
This converse is false! Give an example of a bounded sequence that does not converge.

7. Give an example of a sequence that has both convergent and divergent subsequences.

8. Prove or provide a counterexample: If $\{a_n\}$ and $\{b_n\}$ are bounded, then $\{a_n + b_n\}$ is bounded.

9. Prove that if a sequence $\{a_n\}$ is bounded and monotonic, then $\{a_n\}$ converges.

10. Prove that a sequence having at least one convergent subsequence has infinitely many convergent subsequences. (Hint: Use the theorem which says if $a_n \to a$, then every subsequence of $\{a_n\}$ converges to a.)